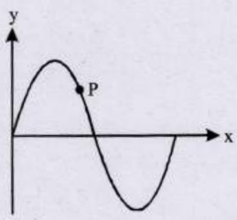
Chapter - Waves

Topic-1: Basic of Mechanical Waves, Progressive and Stationary Waves

1 MCQs with One Correct Answer

1. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap–shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is– **[2008]**



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$

(b) $-\frac{\sqrt{3}\pi}{50}\hat{j}\ m/s$

(c) $\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$

(d) $-\frac{\sqrt{3}\pi}{50}\hat{i}\ m/s$

2. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by **[2000S]**

(a) $\sqrt{\frac{m_1}{m_2}}$ (b) $\sqrt{\frac{m_2}{m_1}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

3. A travelling wave in a stretched string is described by the equation $y = A\sin(kx - \omega t)$ The maximum particle velocity is **[1997 - 1 Mark]**

(a) $A\omega$ (b) ω / k
(c) $d\omega / dk$ (d) x / t

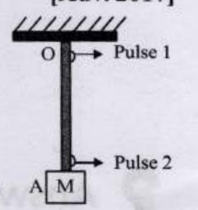
4 Fill in the Blanks

4. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ seconds, where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is .... m/s. **[1990 - 2 Marks]**

5. A travelling wave has the frequency n and the particle displacement amplitude A. For the wave the particle velocity amplitude is ---------- and the particle acceleration amplitude is ---------- . **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

6. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct? **[Adv. 2017]**



(a) The time $T_{AO} = T_{OA}$
(b) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(c) The wavelength of Pulse 1 becomes longer when it reaches point A
(d) The velocity of any pulse along the rope is independent of its frequency and wavelength

7. As a wave propagates, **[1999 - 3 Marks]**
(a) the wave intensity remains constant for a plane wave
(b) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(c) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(d) total intensity of the spherical wave over the spherical surface centred at the source remains constant at all times.

8. In a wave motion $y = a\sin(kx - \omega t)$, y can represent **[1999 - 3 Marks]**
(a) electric field (b) magnetic field
(c) displacement (d) pressure

9. $y(x, t) = 0.8/[4x+5t)^2+5]$ represents a moving pulse, where x and y are in meter and t in second. Then **[1999 - 3 Marks]**

(a) pulse is moving in $+x$ direction
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a sysmmetric pulse

10. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$m and $v = 10$ m s^{-1}, then λ and f are given by **[1998 - 2 Marks]**

(a) $\lambda = 2\pi \times 10^{-2}$ m (b) $\lambda = 10^{-3}$ m
(c) $f = 10^3\ Hz/(2\pi)$ (d) $f = 10^4\ Hz$

11. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in meters and t is in seconds. The expression represents : **[1990 - 2 Marks]**

(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s.
(b) a wave traveling in the negative x-direction with a velocity 1.5 m/s.
(c) a wave travelling in the negative x-direction having a wavelength 0.2 m.
(d) a wave travelling in the positive x-direction having a wavelength 0.2 m.

12. The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are : **[1987 - 2 Marks]**

(a) $\cos kx \sin \omega t$ (b) $k^2x^2 - \omega^2 t^2$
(c) $\cos^2(kx + \omega t)$ (d) $\cos(k^2x^2 - \omega^2 t^2)$

13. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4}\sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave **[1982 - 3 Marks]**

(a) travelling with a velocity of 30 m/s in the negative x direction
(b) of wavelength πm
(c) of frequency $30/\pi$ hertz
(d) of amplitude 10^{-4} m traveling along the negative x-direction

Topic-2: Vibration of String and Organ Pipe

1 MCQs with One Correct Answer

1. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is **[2012]**

(a) 14.0 cm (b) 15.2 cm (c) 16.4 cm (d) 17.6 cm

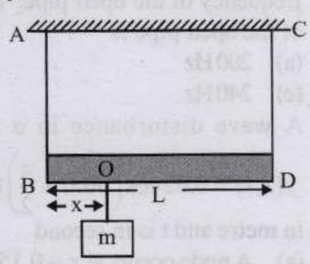
2. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is **[2010]**

(a) 5 grams (b) 10 grams
(c) 20 grams (d) 40 grams

3. In the experiment to determine the speed of sound using a resonance column, **[2007]**

(a) prongs of the tuning fork are kept in a vertical plane
(b) prongs of the tuning fork are kept in a horizontal plane
(c) in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
(d) in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air

4. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is **[2006 - 3M, –1]**



(a) $\frac{L}{5}$
(b) $\frac{4L}{5}$
(c) $\frac{3L}{4}$
(d) $\frac{L}{4}$

5. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option **[2005S]**

(a) $n = 3,\ f_2 = \frac{3}{4}f_1$ (b) $n = 3,\ f_2 = \frac{5}{4}f_1$
(c) $n = 5,\ f_2 = \frac{3}{4}f_1$ (d) $n = 5,\ f_2 = \frac{5}{4}f_1$

6. In a resonance tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is **[2005S]**

(a) 51.2 cm/s (b) 102.4 cm/s
(c) 204.8 cm/s (d) 153.6 cm/s

7. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal **[2004S]**

(a) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (b) $\frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$ (c) $\ell_1\sqrt{\frac{\rho_2}{\rho_1}}$ (d) $\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

8. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **[2002S]**

(a) 25 kg (b) 5 kg
(c) 12.5 kg (d) 1/25 kg

9. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental nodes, the one of length L with frequency ν_1 and the other with frequency ν_2. The raio ν_1/ν_2 is given by **[2000S]**

(a) 2 (b) 4
(c) 8 (d) 1

10. An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is **[1996 - 2 Marks]**

(a) 200 Hz (b) 300 Hz
(c) 240 Hz (d) 480 Hz

11. A wave disturbance in a medium is described by $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$ where x and y are in metre and t is in second **[1995S]**

(a) A node occurs at x = 0.15 m
(b) An antinode occurs at x = 0.3 m
(c) The speed wave is 5 ms^{-1}
(d) The wave length is 0.3 m

12. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **[1995S]**

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$

13. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is **[1988 - 1 Mark]**

(a) $a\sin(kx + \omega t)$ (b) $-a\cos(kx - \omega t)$
(c) $-a\cos(kx + \omega t)$ (d) $-a\sin(kx - \omega t)$

14. A cylindrical tube open at both ends, has a fundamental frequency 'f' in air. The tube is dipped vertically in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column in now **[1981- 2 Marks; Similar JEE Main 2016]**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) f (d) $2f$

15. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is **[1988 - 2 Marks]**

(a) 8/3 (b) 3/8
(c) 1/6 (d) 1/3

16. A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz) **[1986 - 2 Marks]**

(a) 1024 (b) 512 (c) 256 (d) 128

17. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is : **[1985 - 2 Marks]**

(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

2 Integer Value Answer

18. A string of length 1 m and mass 2×10^{-5} kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is ____ Newton. **[Adv. 2023]**

19. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. **[2009]**

3 Numeric Answer

20. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is ____. **[Adv. 2020]**

4 Fill in the Blanks

21. A cylinder resonance tube open at both ends has fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is.................. **[1992 - 1 Mark]**

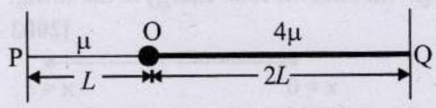
22. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire.

The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ....Hz. **[1987 - 2 Marks]**

23. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ..... metres. **[1984- 2 Marks]**

6 MCQs with One or More than One Correct Answer

24. Two uniform strings of mass per unit length μ and 4μ, and length L and 2L, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is(are) correct? **[Adv. 2024]**



(a) With a node at O, the minimum frequency of vibration of the composite string is v_0.
(b) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.
(c) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.
(d) No vibrational mode with an antinode at O is possible for the composite string.

25. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 *Hz* is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 *cm* and 83.9 *cm*. Which of the following statements is (are) true? **[Adv. 2017]**
(a) The speed of sound determined from this experiment is 332 ms^{-1}
(b) The end correction in this experiment is 0.9 *cm*
(c) The wavelength of the sound wave is 66.4 *cm*
(d) The resonance at 50.7 *cm* corresponds to the fundamental harmonic

26. One end of a taut string of length 3 m along the *x*-axis is fixed at $x = 0$. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the *y*-direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) **[Adv. 2014]**
(a) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$
(b) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$
(c) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$
(d) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

27. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\text{ m})\sin[(62.8\text{ m}^{-1})x]\cos[(628\text{ s}^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) **[Adv. 2013]**
(a) The number of nodes is 5
(b) The length of the string is 0.25 m
(c) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
(d) The fundamental frequency is 100 Hz

28. Standing waves can be produced **[1999 - 3 Marks]**
(a) on a string clamped at both the ends.
(b) on a string clamped at one end free at the other
(c) when incident wave gets reflected from a wall
(d) when two identical waves with a phase difference of π are moving in the same direction

29. The (x, y) co-ordinates of the corners of a square plate are $(0, 0)$, $(L, 0)$, (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant) **[1998 - 2 Marks]**
(a) $a\cos(\pi x/2L)\cos(\pi y/2L)$
(b) $a\sin(\pi x/L)\sin(\pi y/L)$
(c) $a\sin(\pi x/L)\sin(2\pi y/L)$
(d) $a\cos(2\pi x/L)\sin(\pi y/L)$

30. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency : **[1989 - 2 Marks]**
(a) 80 Hz (b) 240 Hz
(c) 320 Hz (d) 400 Hz

7 Match the Following

31. A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range

L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List-I gives the above four strings while list-II lists the magnitude of some quantity. **[Adv. 2019]**

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(a) I → Q, II → P, III → R, IV → T
(b) I → Q, II → S, III → R, IV → P
(c) I → P, II → R, III → S, IV → Q
(d) I → P, II → Q, III → T, IV → S

32. A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. **[Adv. 2019]**

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

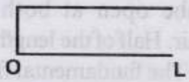
The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$, and $\frac{7L_0}{4}$, respectively. Strings 1, 2, 3, and 4 are vibrated at their 1st, 3rd, 5th, and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

(a) I→T, II→Q, III→R, IV→U
(b) I→P, II→Q, III→T, IV→U
(c) I→P, II→Q, III→R, IV→T
(d) I→P, II→R, III→T, IV→U

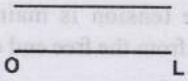
33. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. **[2011]**

Column I / **Column II**

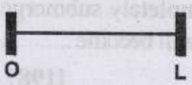
(A) Pipe closed at one end — (p) Longitudinal waves



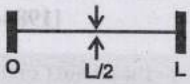
(B) Pipe open at both ends — (q) Transverse waves



(C) Stretched wire clamped at both ends — (r) $\lambda_f = L$



(D) Stretched wire clamped at both ends and at mid-point — (s) $\lambda_f = 2L$



(t) $\lambda_f = 4L$

10 Subjective Problems

34. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[2005 - 4 Marks]**

35. A string tied between $x = 0$ and $x = \ell$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[2003 - 4 Marks]**

36. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. **[2003 - 2 Marks]**

37. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[2002 - 5 Marks]**

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

38. A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height $H (\approx 3.6$ m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurance of first two resonances. Speed of sound in air is 340 m/s and $g = 10$ m/s^2. **[2000 - 10 Marks]**

39. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 m s^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation. **[1998 - 8 Marks]**

(a) Find the length L of the air column.

(b) What is the amplitude of pressure variation at the middle of the column?

(c) What are the maximum and minimum pressures at the open end of the pipe?

(d) What are the maximum and minimum pressures at the closed end of the pipe?

40. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation—

$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ **[1985 - 6 Marks]**

Where x and y are in cm and t in seconds.

(i) What is the maximum displacement of a point at $x = 5$ cm?

(ii) Where are the nodes located along the string?

(iii) What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ sec.?

(iv) Write down the equations of the component waves whose superposition gives the above wave

41. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/sec. Estimate the diameter of the tube. **[1980]**

One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.

Topic-3: Beats, Interference and Superposition of Waves

1 MCQs with One Correct Answer

1. A vibrating string of certain length ℓ under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is **[2008]**

(a) 344 (b) 336 (c) 117.3 (d) 109.3

2. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is **[1998 - 2 Marks]**

(a) 0.05 s (b) 0.10 s (c) 0.20 s (d) 0.40 s

3. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$

This expression may be considered to be a result of the superposition of **[1992 - 2 Marks]**

(a) two (b) three (c) four (d) five

2 Integer Value Answer

4. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **[2010]**

Topic-4: Musical Sound and Doppler's Effect

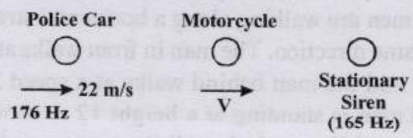
1 MCQs with One Correct Answer

1. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **[2011]**

(a) 8.50 kHz (b) 8.25 kHz

(c) 7.75 kHz (d) 7.50 kHZ

2. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats. **[2003S]**



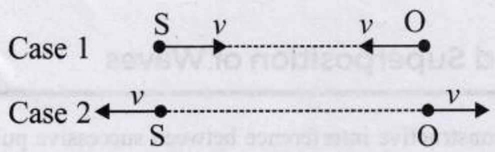
(a) 33m/s (b) 22m/s

(c) zero (d) 11m/s

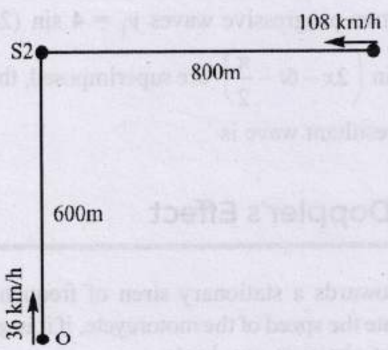
3. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is **[2002S]**
(a) 242/252 (b) 2
(c) 5/6 (d) 11/6

2 Integer Value Answer

4. A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is **[Adv. 2024]**



5. A train S1, moving with a uniform velocity of 108 km/h approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36km/h towards S2 as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600m away from S2 and distance between S1 and S2 is 800m, the number of beats heard by O is ______
[Speed of the sound = 330m/s] **[Adv. 2019]**



6. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is ______ **[Adv. 2018]**

7. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original.

What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received). **[Adv. 2017]**

8. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is **[Adv. 2015]**

9. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. **[2010]**

3 Numeric Answer

10. A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1}, the smallest value of the percentage change required in the length of the pipe is ______. **[Adv. 2020]**

4 Fill in the Blanks

11. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (Speed of sound in air = 342 ms^{-1}) **[1994 - 2 Marks]**

5 True / False

12. A source of sound with frequency 256 Hz is moving with a velocity V towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats **[1985 - 3 Marks]**

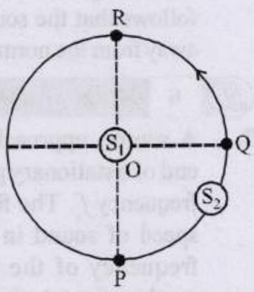
13. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. **[1980]**

8 Comprehension Based Questions

Directions for questions no. 14 and 15:

PARAGRAPH "I"

S_1 and S_2 are two identical sound sources of frequency 656 Hz. The source S_1 is located at O and S_2 moves anti-clockwise with a uniform speed $4\sqrt{2}\ ms^{-1}$ on a circular path around O, as shown in the figure. There are three points P, Q and R on this path such that P and R are diametrically opposite while Q is equidistant from them. A sound detector is placed at point P. The source S_1 can move direction OP.



[Given: The speed of sound in air is $324\ m\ s^{-1}$]

14. When only S_2 is emitting sound and it is at Q, the frequency of sound measured by the detector in Hz is ________ **[Adv. 2023]**

15. Consider both sources emitting sound. When S_2 is at R and S_1 approaches the detector with a speed $4\ m\ s^{-1}$, the beat frequency measured by the detector is ________ Hz. **[Adv. 2023]**

16. Each of the properties of sound listed in the column *A* primarily depends on one of the quantities in column *B*. Write down the matching pairs from the two columns. **[1980]**

Column A	Column B
(A) pitch	(p) Waveform
(B) quality	(q) frequency
(C) loudness	(r) intensity

10 Subjective Problems

17. A boat is traveling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

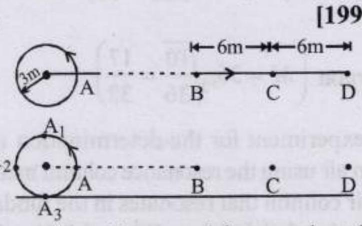
(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C; Density of river water = $10^3\ kg/m^3$; Bulk modulus of the water = 2.088×10^9 Pa; Gas constant $R = 8.31$ J/mol-K; Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4) **[2001 - 10 Marks]**

18. A band playing music at a frequency *f* is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If *v* is the speed of sound, obtain an expression for the beat frequency heard by the motorist. **[1997 - 5 Marks]**

19. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. **[1996 - 3 Marks]**

20. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line *BD* with an amplitude $BC = CD = 6$ metres. The frequency of oscillation of the detector is $\frac{5}{\pi}$ per second. The source is at the point *A* when the detector is at the point *B*. If the source emits a continous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. **[1990 - 7 Mark]**



21. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. **[1986 - 8 Marks]**

22. A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. **[1982 - 7 Marks]**

23. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? **[1981 - 4 Marks]**

Topic-5: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. A student is performing an experiment using a resonance column and a tuning fork of frequency $244\ s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is
(Useful information: $\sqrt{167RT} = 640 J^{1/2} mole^{-1/2}$; $\sqrt{140RT} = 590 J^{1/2} mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.) **[Adv. 2014]**

(a) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(b) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

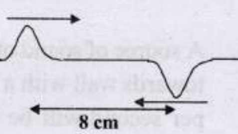
(c) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(d) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

2. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction. **[2003S]**
(a) 0.012m (b) 0.025m (c) 0.05m (d) 0.024m

3. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then **[2001S]**
(a) $E_2 = E_1$ (b) $E_2 = 2E_1$ (c) $E_2 = 4E_1$ (d) $E_2 = 16E_1$

4. Two pulses in a stretched string whose centers are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be **[2001S]**
(a) zero
(b) purely kinetic
(c) purely potential
(d) partly kinetic and partly potential



5. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be **[1996 - 2 Marks]**
(a) 1.22v (b) 0.61v (c) 1.50v (d) 0.75v

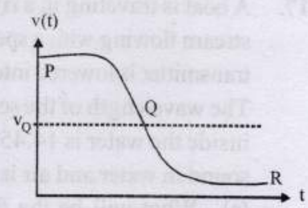
5 True / False

6. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **[1984- 2 Marks]**

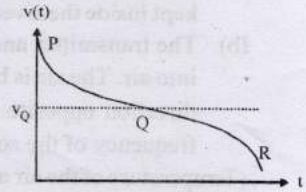
6 MCQs with One or More than One Correct Answer

7. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance? **[Adv. 2021]**
(a) $u = 0.8\,v$ and $f_s = f_0$ (b) $u = 0.8\,v$ and $f_s = 2f_0$
(c) $u = 0.8\,v$ and $f_s = 0.5 f_0$ (d) $u = 0.5\,v$ and $f_s = 1.5\, f_0$

8. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person? **[Adv. 2016]**
(a) $v_P + v_R = 2\,v_Q$
(b) The rate of change in beat frequency is maximum when the car passes through Q
(c) The plot below represents schematically the variation of beat frequency with time



(d) The plot below represents schematically the variation of beat frequency with time



9. A person blows into open-end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe, **[2012]**

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.
(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.
(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

10. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then, **[2009]**
(a) the intensity of the sound heard at the first resonance was more than that at the second resonance
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(d) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

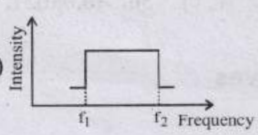
8 Comprehension/Passage Based Questions

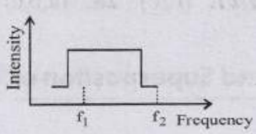
Passage - 1

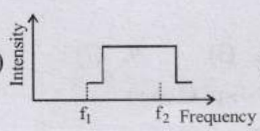
Two trains A and B moving with speeds 20 m/s and 30 m/s respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.
Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s. **[2007]**

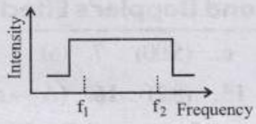
11. The speed of sound of the whistle is
(a) 340 m/s for passengers in A and 310 m/s for passengers in B
(b) 360 m/s for passengers in A and 310 m/s for passengers in B
(c) 310 m/s for passengers in A and 360 m/s for passengers in B
(d) 340 m/s for passengers in both the trains

12. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by

(a) 

(b) 

(c) 

(d) 

13. The spread of frequency as observed by the passengers in train B is
(a) 310 Hz (b) 330 Hz (c) 350 Hz (d) 290 Hz

Passage - 2

Waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling along x-axis. (Here x is in m and t is in second) **[2006 – 5M, –2]**

14. Find the number of times intensity is maximum in time interval of 1 sec.
(a) 4 (b) 6 (c) 8 (d) 10

15. The wave velocity of louder sound is
(a) 100 m/s (b) 192 m/s (c) 200 m/s (d) 96 m/s

16. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 sec is
(a) 100 (b) 46 (c) 192 (d) 96

10 Subjective Problems

17. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2A^2$. **[1993 - 4 Marks]**
(i) Find the time interval between successive maxima of the intensity of the signal received by the detector.
(ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

18. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. **[1991 - 4 × 2 Marks]**
(a) What are the wavelength and frequency of incident wave?
(b) Write the equation for the reflected wave.
(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.
(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave ? What is the direction of propagation of travelling wave?

19. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train Find **[1988 - 5 Marks]**
(i) the Frequency of the whistle as heard by an observer on the hill,
(ii) the distance from the hill at which the echo from the hill is heard by the driver and its frequency.
(Velocity of sound in air = 1,200 km/hr)

20. The following equations represent transverse waves : **[1987 - 7 Marks]**
$z_1 = A\cos(kx - \omega t)$;
$z_2 = A\cos(kx + \omega t)$; $z_3 = A\cos(ky - \omega t)$
Identify the combination (s) of the waves which will produce (i) standing wave (s), (ii) a wave travelling in the

directon making an angle of 45° degrees with the positive x and positive y axes. In each case, find the positions at which the resultant intensity is always zero.

21. A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area $10^{-6}\ m^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by 20° C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration.

Given for steel $Y = 2\times10^{11}\ N/m^2$

$\alpha = 1.21\times10^{-5}$ per °C **[1984 - 6 Marks]**

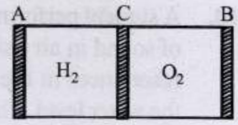
22. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

Given : Young modulus of copper $= 1.3\times10^{11}\ N/m^2$.

Coefficient of linear expansion of copper $= 1.7\times10^{-5}\ °C^{-1}$.

Density of copper $= 9\times10^3\ kg/m^3$. **[1979]**

23. AB is a cylinder of length 1m fitted with a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[1978]**



Answer Key

Topic-1 : Basic of Mechanical Waves, Progressive and Stationary Waves

1. (a) **2.** (b) **3.** (a) **4.** (0.5) **5.** $(A2\pi\nu, 4\pi^2\nu^2A)$ **6.** (a, d) **7.** (a, c, d) **8.** (a,b, c,d)
9. (b,c,d) **10.** (a, c) **11.** (b, c) **12.** (a, c) **13.** (a,b, c,d)

Topic-2 : Vibration of String and Organ Pipe

1. (b) **2.** (b) **3.** (a) **4.** (a) **5.** (d) **6.** (c) **7.** (b) **8.** (a) **9.** (d) **10.** (a)
11. (c) **12.** (a) **13.** (c) **14.** (c) **15.** (c) **16.** (a) **17.** (a, c) **18.** (5) **19.** (5)
20. (0.62 to 0.63) **24.** (a,c,d) **25.** (a,b,c) **26.** (a, c, d) **27.** (b, c) **28.** (a,b,c) **29.** (b, c) **30.** (a,b,d) **31.** (c)
32. (b) **33.** A-p,t; B-p,s; C-q,s; D-q,r

Topic-3 : Beats, Interference and Superpositon of Waves

1. (a) **2.** (b) **3.** (b) **4.** (5)

Topic-4 : Musical Sound and Doppler's Effect

1. (a) **2.** (b) **3.** (b) **4.** (200) **5.** (8.13) **6.** (5.00) **7.** (6) **8.** (3) **9.** (7)
10. (0.62 to 0.63) **12.** (False) **13.** (False) **14.** (648) **15.** (8.2) **16.** $(A\to q; B\to p; C\to r)$

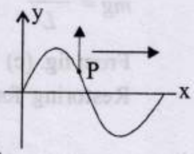
Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (d) **2.** (b) **3.** (c) **4.** (b) **5.** (a) **7.** (a,d) **8.** (a,b,c) **9.** (b,d) **10.** (a,d)
11. (b) **12.** (a) **13.** (a) **14.** (a) **15.** (c) **16.** (d)

Hints & Solutions

Topic-1: Basic of Mechanical Waves, Progressive and Stationary Waves

1. **(a)** Particle velocity v_p is related to the displacement of the particle from the mean position as



$$v_p = 2\pi v\sqrt{A^2 - y^2}$$

$$v_p = 2\pi\left(\frac{v}{\lambda}\right)\sqrt{A^2 - y^2}$$

$$= \frac{2\pi}{0.5} \times 0.1\sqrt{(0.1)^2 - (0.05)^2} = \frac{\sqrt{3}\pi}{50}\hat{j} \text{ m/s}$$

Since the wave is sinusoidal moving in positive x-axis the point will move parallel to y-axis therefore options (c) and (d) are ruled out. As the wave moves forward in positive X-direction, the point should move upwards i.e. in the positive Y-direction.

2. **(b)** $V = \sqrt{\frac{\gamma RT}{M_0}}$, where M_0 = molecular mass

So, $V \propto \sqrt{\frac{1}{M_0}} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{M_{02}}{M_{01}}} = \sqrt{\frac{m_2}{m_1}}$

3. **(a)** $v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t) \quad \therefore \quad v_{max} = A\omega$

Maximum particle velocity of the particle in SHM, $v_{max} = A\omega$

4. **(0.5)** Given : $y = \frac{1}{(1+x)^2}$

At $t = 0$ we get $y = 1$ when $x = 0$

Again, $y = \frac{1}{1+(x-1)^2}$

So, at $t = 2$ $y = 1$ when $x = 1$

The wave pulse has travelled a distance of 1m in 2 sec.

$\therefore$ Velocity of wave pulse, $v = \frac{1}{2} = 0.5\,\text{ms}^{-1}$

5. **($A2\pi v$, $4\pi^2 v^2 A$)** Particle velocity amplitude $= V_{max} = A\omega = A2\pi v$

Particle acceleration amplitude $= a_{max} = A\omega^2 = 4\pi^2 v^2 A$

6. **(a, d)** Wavelength of pulse, $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$ or, $T \propto \sqrt{T}$

Where T = tension of string.

Here $T_1 > T_2 \quad \therefore \lambda_1 > \lambda_2$

The velocities of the two pulses cannot be same at mid-point as velocity being vector quantity has direction.

$V = \sqrt{\frac{T}{\mu}}$, so speed at any position will be same for both pulses, therefore time taken by both pulses will be same i.e., $T_{AO} = T_{OA}$

7. **(a, c, d)** For a plane wave, intensity i.e., energy crossing per unit area per unit time is constant at all points.

But for a spherical wave, intensity at a distance r from a point source

$I \propto \frac{1}{r^2}$

But the **total intensity** of the spherical wave over the spherical surface centered at the source remains constant at all times.

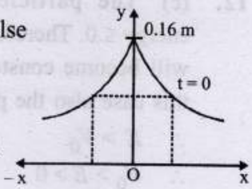
For line source $I \propto \frac{1}{r}$ spherical wave is not produced by the line source.

8. **(a, b, c, d)** In the wave motion $y = a(kx - \omega t)$, y can represent, electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

9. **(b, c, d)** Comparing the given equation, $y(x, t)$

$$= \frac{0.8}{(4x+5t)^2 + 5} = \frac{0.8}{16\left[x + \frac{5}{4}t\right]^2 + 5}$$

With the equation of moving pulse $y = f(x + vt)$



$v = \frac{5}{4}\text{ms}^{-1} = \frac{2.5}{2}\text{ms}^{-1}$

So, the wave will travel a distance of 2.5 m in 2 sec.

At $x = 0, t = 0$, $y = \frac{0.8}{(4x+5t)^2 + 5} = \frac{0.8}{5} = 0.16$ m

$\therefore$ Maximum displacement is 0.16 m

The shape of the pulse at $x = 0$ and $t = 0$ is as shown in figure and it is symmetric.

10. **(a, c)** For a transverse sinusodial wave travelling on a string, the maximum velocity $v_{max} = a\omega$.

Given maximum velocity $= \frac{v}{10} = \frac{10}{10} = 1$ m/s

$\therefore \quad a\omega = 1 \quad \Rightarrow \quad 10^{-3} \times 2\pi v = 1 \; [\because \omega = 2\pi v]$

$\Rightarrow \quad v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

And, $\lambda = \frac{v}{v} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}$ m

11. **(b, c)** Given equation, $y = A\sin(10\pi x + 15\pi t + \pi/3)$

Comparing this equation with standard equation of a wave travelling in $-X$ direction

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right] \Rightarrow y = A\sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$$

$\frac{2\pi v}{\lambda} = 15\pi$ and $\frac{2\pi}{\lambda} = 10\pi$

$\Rightarrow \lambda = \frac{1}{5} = 0.2$ m and $v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5$ m/s

12. **(a,c)** For wave motion, the differential equation is

$$\frac{\partial^2 y}{\partial t^2} = \left(\text{constant } \frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2} \quad \left[\because v = \frac{\omega}{k}\right]$$

or $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ....(i)

The wave motion is characterized by the two conditions

$f(x, t) = f(x, t + T)$ ....(ii)

$f(x, t) = f(x + \lambda, t)$ ....(iii)

13. **(a,b,c,d)** Given: $y = 10^{-4}\sin(60t + 2x)$
Comparing the given equation with the standard wave equation $y = a\sin(\omega t + kx)$
Amplitude $a = 10^{-4}$m; $k = 2\text{ m}^{-1}$

And, $\omega = 60$ rad/s $\Rightarrow 2\pi f = 60 \quad \therefore \quad f = \frac{30}{\pi}$Hz

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ m

Speed of wave $v = \frac{\omega}{k} = \frac{60}{2} = 30$ m/s

Topic-2: Vibration of String and Organ Pipe

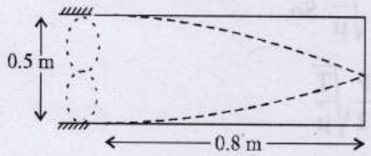
1. **(b)** Considering the end correction [$e = 0.3\,D$ where D = diameter]

$f = n\left[\frac{v}{4(l+e)}\right]$ For first resonance, $n = 1$

$\therefore f = \frac{v}{4(l+0.3D)} \Rightarrow l = \frac{v}{4f} - 0.3D$

or, $l = \left(\frac{336\times100}{4\times512}\right) - 0.3\times4 = 15.2$cm

2. **(b)** Frequency of 2nd harmonic of string = fundamental frequency produced in the pipe

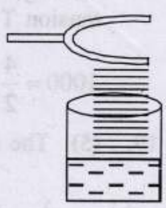


$2\left(\frac{v_1}{2l_1}\right) = \frac{v_2}{4l_2} \quad \therefore 2\times\left[\frac{1}{2l_1}\sqrt{\frac{T}{\mu}}\right] = \frac{v}{4l_2}$

$\therefore \frac{1}{0.5}\sqrt{\frac{50}{\mu}} = \frac{320}{4\times0.8} \Rightarrow \mu = 0.02\text{ kg m}^{-1}$

Hence, the mass of the string, $m_1 = \mu\, l_1 = 0.02\times0.5\text{ kg} = 10$g

3. **(a)** To determine the speed of sound using a resonance column, prongs of the tuning fork are kept in a verticle plane. As shown in the figure, the fringes of the tuning fork are kept in a vertical plane.



4. **(a)** Frequency of first harmonic in AB $= f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

Frequency of second harmonic in CD $= f_2 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$f_1 = f_2$ (given)

$\therefore \frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{1}{l}\sqrt{\frac{T_2}{\mu}}$ or $T_1 = 4T_2$ ...(i)

Equating torques due to T_1 and T_2 about O for rotational equilibrium.

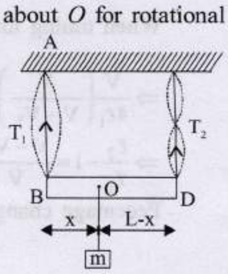
$\therefore T_1 x = T_2(L - x)$

For translational equilibrium,

$T_1 + T_2 = mg$

From eg. (i) and (iii),

$T_1 = \frac{4mg}{5}$ and $T_2 = \frac{mg}{5}$



Now from eq. (ii).

$\frac{4mg}{5}\times x = \frac{mg}{5}(L-x) \Rightarrow 4x = L - x \therefore x = \frac{L}{5}$.

5. **(d)** Frequency of 2nd harmonic of open pipe,

$f_1 = \frac{v}{\lambda} = \frac{v}{\ell}$ ... (i)

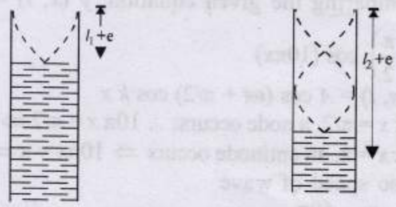
Frequency of nth harmonic of closed pipe,

$f_2 = \frac{v}{\lambda} = \frac{nv}{4\ell}$ ... (ii)

Here n is a odd number. From eq. (i) and (ii)

$f_2 = \frac{n}{4}f_1$, For first resonance, $n = 5 \quad \therefore f_2 = \frac{5}{4}f_1$

6. **(c)** In a resonance tube,



For first resonance **For second resonance**

$\ell_1 + e = \frac{\lambda}{4}$ $\quad\quad \ell_2 + e = \frac{3\lambda}{4}$

But $v = \nu\lambda$

$\therefore \quad v = \nu\frac{4}{3}(\ell_2 + e) \quad \Rightarrow \ell_2 + e = \frac{3v}{4\nu}$ ...(i)

$\therefore \quad v = \nu\, 4(\ell_1 + e) \quad \Rightarrow \ell_1 + e = \frac{v}{4\nu}$ ... (ii)

Subtracting (i) and (ii),

$v = 2\nu(\ell_2 - \ell_1) \quad \therefore \Delta v = 2\nu(\Delta\ell_2 + \Delta\ell_1)$
$= 2\times512\times(0.1 + 0.1)$ cm/s $= 204.8$ cm/s

Hence, maximum possible error in speed, $\Delta v = 204.8$ cm/s

7. **(b)** Frequency of first overtone in closed organ pipe,

$\nu = \frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}}$

Frequency of first overtone in open organ pipe,

$\nu' = \frac{1}{\ell_2}\sqrt{\frac{P}{\rho_2}}$

Here, $\nu = \nu'$

$\frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}} = \frac{1}{l_2}\sqrt{\frac{P}{\rho_2}} \quad \therefore \quad \ell_2 = \frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$

8. **(a)** Fundamental frequency, $f_0 = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

$= \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}} \Rightarrow M = 25$ kg

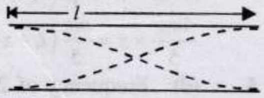
As frequency is corresponds to 5th and 3rd harmonic $P = 5$ and $P = 3$ respectively.

9. **(d)** $n_1 = \frac{1}{2\ell}\sqrt{\left(\frac{T}{4\pi r^2\rho}\right)}$ and $n_2 = \frac{1}{4\ell}\sqrt{\left(\frac{T}{\pi r^2\rho}\right)}$

$n = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$ [where $\frac{\lambda}{2}$ = length of string]

$\therefore \frac{n_1}{n_2} = 2\times\frac{1}{2} = 1 \left[\because m = \frac{\text{mass}}{\text{length}} = \frac{\rho\times A\times\text{length}}{\text{length}} = \rho A\right]$

10. (a) **For both end open organ pipe**
Fundamental frequency



$\nu_1 = \frac{c}{2\ell}$ ...(i)

For one end closed organ pipe
For third harmonic frequency

$\nu_2 = 3\left(\frac{c}{4l}\right)$ ... (ii)

Given $\nu_2 - \nu_1 = 100$ ... (iii)

From eq. (i) and (ii)

$\frac{\nu_2}{\nu_1} = \frac{3/4}{1/2} = \frac{3}{2} \Rightarrow \nu_1 = \frac{2}{3}\nu_2$ ...(iv)

Again, solving eq. (iii) & (iv) we get, $\nu_1 = 200$ Hz.

11. (c) Comparing the given equation, $y(x, t) = 0.02 \cos\left(50\pi t + \frac{\pi}{2}\right) \cos(10\pi x)$

$y(x, t) = A \cos(\omega t + \pi/2) \cos kx$

If $kx = \pi/2$, a node occurs; $\therefore 10\pi x = \pi/2 \Rightarrow x = 0.05$ m

If $kx = \pi$, an antinode occurs $\Rightarrow 10\pi x = \pi \Rightarrow x = 0.1$ m

Also speed of wave

$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s and $\lambda = 2\pi/k = 2\pi/10\pi = 0.2$ m

12. (a) *In air* : $T = mg = \rho V g$

$\therefore f = \sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{\rho V g}{m}}$ ... (i)

when the object is half immersed in water.

$T = mg -$ upthrust

$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$

$\therefore f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}}$

$\frac{f'}{f} = \sqrt{\frac{2\rho-\rho_\omega}{2\rho}} \quad f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2} = 300\left[\frac{2\rho-1}{2\rho}\right]^{1/2}$ Hz

13. (c) Stationary wave is produced when two waves travel in opposite direction.

Now, $y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$

$\therefore$ $y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.

$y = -a\cos(kx - \omega t)$ cannot be as their directions are not opposite.

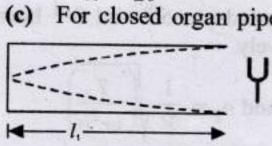
14. (c) When tube was open at both ends, then $\frac{\lambda}{2} = \ell$

$\therefore f = \frac{v}{\lambda} = \frac{v}{2\ell}$

when tube is half dipped in water, then $\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$

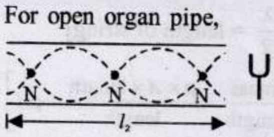
$\therefore f' = \frac{v}{\lambda'} = \frac{v}{2\ell} = f$

15. (c) For closed organ pipe,



$\nu_1 = \frac{v}{4\ell_1}$ for first harmonic

For open organ pipe,



$\nu_2 = \frac{3v}{2\ell_2}$ for third harmonic

$\because \nu_1 = \nu_2 \quad \therefore \frac{v}{4\ell_1} = \frac{3v}{2\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{1}{6}$

16. (a) For tube closed at are end

$\frac{\lambda}{4} = \ell$ (Fundamental mode) $\therefore \lambda = 4\ell$

For open tube

$\frac{\lambda'}{2} = \ell$ (Fundamental mode) $\therefore \lambda' = 2\ell$

$\therefore \nu = \frac{c}{\lambda} = \frac{c}{4\ell} = 512$ Hz (given)

and $\nu' = \frac{c}{\lambda'} = \frac{c}{2\ell} = 2\left(\frac{c}{4\ell}\right) = 2 \times 512 = 1024$ Hz.

17. (a, c) For closed organ pipe,

$f = n\left(\frac{v}{4l}\right)$ where, $n = 1, 3, 5......$ $\therefore l = \frac{nv}{4f}$

For $n = 1$, $l_1 = \frac{(1)(330)}{4 \times 264} \times 100$ cm $= 31.25$ cm

For $n = 3$, $l_3 = 3l_1 = 93.75$ cm

For $n = 5$, $l_5 = 5l_1 = 156.25$ cm

18. (5) As two successive harmonics are found to occur at frequency 750 Hz and 1000Hz and frequency

$f = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$ So,

$750 = \frac{P}{2}\sqrt{\frac{T}{\mu}}$ ......(i)

and, $1000 = \frac{P+1}{2}\sqrt{\frac{T}{\mu}}$ ......(ii)

Dividing eq (ii) by (i)

$\frac{4}{3} = \frac{P+1}{P} \quad \therefore P = 3$

Putting this value of P = 3 in eq. (ii) and solving we get tension T

$1000 = \frac{4}{2}\sqrt{\frac{T}{2\times 10^{-5}}} \quad \therefore T = 5N$

19. (5) The distance between two successive nodes

$D = \frac{\lambda}{2} = \frac{v}{2f} = \frac{\sqrt{T/\mu}}{2f} = \frac{\sqrt{\frac{0.5\times 0.2}{10^{-3}}}}{2\times 100} = \frac{10}{2} = 5$cm

20. (0.62 to 0.63)

Let ℓ_1 = initial length of pipe

ℓ_2 = new length of pipe

V_T = Speed of tuning fork

In closed organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f = \left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2}$

$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V-V_T}\right) = \frac{V}{4\ell_2} \quad \Rightarrow \frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$

$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1 \Rightarrow \frac{\ell_2-\ell_1}{\ell_1} = \frac{-V_T}{V}$

Pecentage change required in the length of the pipe

$$\frac{\ell_2-\ell_1}{\ell_1}\times 100=\frac{-2}{320}\times 100=-0.625\%$$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

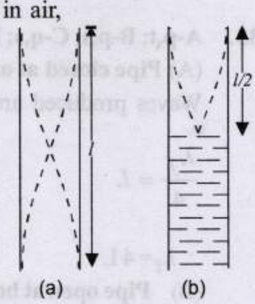
21. In figure (a) both ends of the tube in air,

$$\frac{\lambda}{2}=\ell \Rightarrow \lambda=2\ell \therefore f=\frac{c}{\lambda}=\frac{c}{2\ell}$$

In figure (b) when half of the tube vertically dipped in water,

$$\frac{\lambda'}{4}=\frac{\ell}{2} \Rightarrow \lambda'=2\ell$$

$$\therefore f'=\frac{c}{\lambda'}=\frac{c}{2\ell}=f$$



22. Fundamental frequency $f=\frac{1}{2l}\sqrt{\frac{T}{\mu}}$ or $f\propto\sqrt{T}$

$f\propto\sqrt{\text{weight of mass }(w)}$

$f'\propto\sqrt{\text{w}-\text{upthrust }(F)}$

$$\therefore \frac{f'}{f}=\sqrt{\frac{w-F}{w}} \text{ or, } f'=f\sqrt{\frac{w-F}{w}}$$

Substituting the values, we get

$$f'=260\sqrt{\frac{(50.7)g-(0.0075)(10^3)g}{(50.7)g}}=240Hz$$

23. As $c=v\lambda \therefore \lambda=\frac{c}{v}=\frac{330}{660}=0.5\text{m}$

Shortest distance from the wall at which air particles have maximum amplitude of vibration $=\lambda/4=\frac{0.5}{4}=0.125\text{m}$

24. **(a, c, d)** The velocity of a transverse wave in a stretched string, $C=\sqrt{\frac{T}{\mu}}$

$$C_1=\sqrt{\frac{T}{\mu}}, C_2=\sqrt{\frac{T}{4\mu}}=\frac{C_1}{2}$$

For node at O

$$L=\frac{n\lambda_1}{2} \text{ and } 2L=\frac{m\lambda_2}{2} \text{ (Here, n, m are integers)}$$

$$\text{or, } \lambda_1=\frac{2L}{n} \text{ and } \lambda_2=\frac{4L}{m}$$

$$\frac{C_1}{\lambda_1}=\frac{C_2}{\lambda_2}\Rightarrow\frac{C_1}{\frac{2L}{n}}=\frac{\frac{C_1}{2}}{\frac{4L}{m}}$$

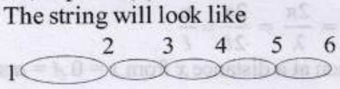
$\therefore \quad 4n=m$

For minimum frequency, n = 1, m = 4

$$\therefore \ v_{min}=\frac{C_1\times 1}{2L}=\frac{1}{2L}\sqrt{\frac{T}{\mu}}=v_0$$

So, option (a) is correct

The string will look like



i.e. 6 nodes including the end nodes so option (c) is correct.

For antinode at O

$$L=(2n+1)\frac{\lambda_1}{4} \text{ and } 2L=(2n+1)\frac{\lambda_2}{4} \text{ (n,m are integers)}$$

$$\text{or, } \lambda_1=\frac{4L}{(2n+1)} \text{ and } \lambda_2=\frac{8L}{(2m+1)}$$

$$m\frac{1}{4L}\sqrt{\frac{T}{\mu}}=n\frac{1}{4(2L)}\sqrt{\frac{T}{4\mu}}$$

$$\Rightarrow \frac{m}{n}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

$$\therefore \ m=1, f_{min}=1\frac{1}{4L}\sqrt{\frac{T}{\mu}}=\frac{v_0}{2}$$

So, option (b) is incorrect.

25. **(a, b, c)** According to question, the length of the air column is varied by changing the level of water in the resonance tube,

so, $(2n+1)\frac{\lambda}{4}=50.7+e$ ...(i)

and $(2n+3)\frac{\lambda}{4}=83.9+e$ ...(ii)

Dividing eq (i) by (ii)

If $n=1$, $\frac{3\lambda/4}{5\lambda/4}=\frac{50.7+e}{83.9+e}$ $\therefore 3\times 83.9+3e=5\times 50.7+5e$

$\Rightarrow \quad 2e=1.8 \qquad \therefore e=0.9$ cm

$\therefore \quad \frac{3\lambda}{4}=50.7+0.9=51.6 \quad \Rightarrow \lambda=66.4\text{ cm}=0.664\text{ m}$

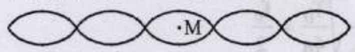
Also speed of sound, $V=v\lambda=500\times 0.664\text{ ms}^{-1}=332.0\text{ ms}^{-1}$

26. **(a, c, d)** There should be a displacement node at $x=0$ and a displacement antinode at $x=3$ m. Therefore, $y=0$ at $x=0$ and $y=\pm A$ at $x=3$ m.

Speed of wave, $v=\frac{\omega}{k}=100\text{ ms}^{-1}$.

Hence options (a), (c) & (d) satisfy the above conditions.

27. **(b, c)** y = [0.01 sin (62.8x)] cos (628 t). [Given]



From the given equation, $k=\frac{2\pi}{\lambda}=62.8 \therefore \lambda=\frac{2\pi}{62.8}=0.1\text{m}$

Length of string, $l=5\times\frac{\lambda}{2}=5\times\frac{1}{20}=0.25$ m

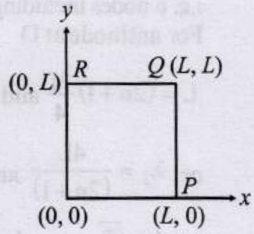
The midpoint M is an antinode and has the maximum displacement = 0.01 m

The fundamental frequency, $v=\frac{v}{2l}=\frac{\omega/k}{2l}$

$$=\frac{628}{2\times 0.25\times 62.8}=20\text{Hz}$$

28. **(a,b,c)** Standing waves are produced by two identical waves superposing while travelling in opposite direction.

29. **(b,c)** The edges of the plate are clamped, so its displacements along the x and y axes will individually be zero at the edges.



Option (a) :
$u(x, y) = 0$ at $x = L, y = L$
$u(x, y) \neq 0$ at $x = 0, y = 0$
Option (b) :
$u(x, y) = 0$ at $x = 0, y = 0 [\because \sin 0 = 0]$
$u(x, y) = 0$ at $x = L, y = L [\because \sin \pi = 0]$
Option (c) :
$u(x, y) = 0$ at $x = 0, y = 0 [\because \sin 0 = 0]$
$u(x, y) = 0$ at $x = L, y = L [\because \sin \pi = 0, \sin 2\pi = 0]$
Option (d) :
$u(x, y) = 0$ at $y = 0, y = L [\because \sin 0 = 0, \sin \pi = 0]$
$u(x, y) \neq 0$ at $x = 0, x = L [\because \cos 0 = 1, \cos 2\pi = 1]$

30. **(a, b, d)** For closed organ pipe, $v = n\left(\frac{v}{4l}\right)$ where n = 1, 3, 5, 7....

$\therefore \quad v = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}, ... = 80, 240, 400....$

31. **(c)** Frequency, $v = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ for first mode of vibration

For 'v' to be maximum, 'ℓ' should be minimum.

String-1 $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

String-2 $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

String-3 $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

String-4 $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

32. **(b)** As $v = \frac{p}{2\ell}\sqrt{\frac{T}{m}} \therefore T = \frac{v^2\ell^2 m}{p^2}$

String-1 $T_0 = \frac{f_0^2 4L_0^2 \mu}{\ell^2}$

String-2 $T_2 = \frac{f_0^2 4\left(\frac{3}{2}\right)^2 L_0^2 (2\mu)}{(3)^2} = \frac{T_0}{2}$

String-3 $T_3 = \frac{f_0^2 4\left(\frac{5}{2}\right)^2 L_0^2 (3\mu)}{5^2} = \frac{3}{16}T_0$

String-4 $T_4 = \frac{f_0^2 4\left(\frac{7}{4}\right)^2 L_0^2 (4\mu)}{(14)^2} = \frac{T_0}{16}$

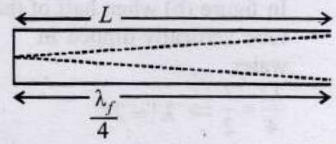
33. **A-p,t; B-p,s; C-q,s; D-q,r**

(A) **Pipe closed at one end**

Waves produced are longitudinal

$\frac{\lambda_f}{4} = L$

$\therefore \lambda_f = 4L$

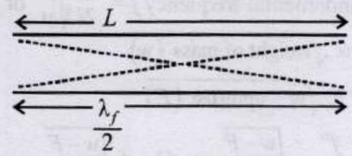


(B) **Pipe open at both ends**

waves produced are longitudinal

$\frac{\lambda_f}{2} = L$

$\therefore \lambda_f = 2L$

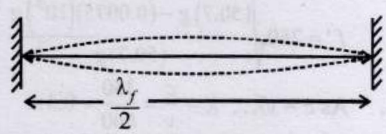


(C) **Stretched wire clamped at both ends**

Waves produced are transverse in nature.

$\frac{\lambda_f}{2} = L$

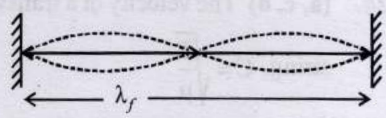
$\therefore \lambda_f = 2L$



(D) **Stretched wave clamped at both ends and at mid point**

Waves produced are transverse in nature

$\lambda_f = L$



34. The standard waveform of a transverse harmonic disturbance

$y = a \sin(\omega t \pm kx \pm \phi)$

Given $v_{max} = a\omega = 3$ m/s ... (i)

$A_{max} = a\omega^2 = 90$ m/s^2 ... (ii)

Velocity of wave $v = 20$ m/s ... (iii)

Dividing eq. (ii) by (i)

$\frac{a\omega^2}{a\omega} = \frac{90}{3} \Rightarrow \omega = 30$ rad/s ... (iv)

Substituting the value of ω in eq. (i), we get

$a = \frac{3}{30} = 0.1$m ... (v)

Now, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/\nu} = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2}$ ... (vi)

Now putting the value of a, ω and k we get waveform

$y = 0.1 \sin [30t \pm \frac{3}{2}x \pm \phi]$

35. The string vibrates in fundamental mode therefore $\ell = \frac{\lambda}{2}$

or $\lambda = 2\ell$ Since, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\ell} = \frac{\pi}{\ell}$

Amplitude of vibration at a distance x from $x = 0$ $A = a \sin kx$

Mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a\sin kx)^2(2\pi\nu)^2$$

$$= 2\pi^2\mu\nu^2a^2\sin^2 kx\, dx$$

$\because v = \nu\lambda$

$$\therefore \quad \nu = \frac{v}{\lambda} \Rightarrow \nu^2 = \frac{v^2}{\lambda^2} = \frac{T/\mu}{4\ell^2} \qquad \left[\because v = \sqrt{T/\mu}\right]$$

$$\therefore \quad dE = 2\pi^2\mu\frac{T/\mu}{4\ell^2}a^2\sin^2\left\{\left(\frac{\pi}{\ell}\right)x\right\}dx$$

$\therefore$ Total energy of the string

$$E = \int dE = \int_0^\ell 2\pi^2\mu\frac{T/\mu}{4\ell^2}a^2\sin^2\left(\frac{\pi x}{\ell}\right)dx$$

$$\text{or, } E = \frac{\pi^2 Ta^2}{4\ell}$$

36. In the fundamental mode

$$(\ell + 0.6r) = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow v = 4f\,(\ell + 0.6r)$$

Here, $0.6\,r$ = end-correction in tube
r (= radius of tube) = 2.5 cm
l (= length of tube) = 16 cm
f (= frequency of tuning fork) = 480 Hz
$\therefore$ Velocity of sound in air, $v = 336$ m/s.

37. (a) Frequency of second harmonic in pipe A open at both ends = frequency of third harmonic in pipe B closed at one end

$$\therefore 2\left(\frac{v_A}{2l_A}\right) = 3\left(\frac{v_B}{4l_B}\right)$$

$$\text{or } \frac{v_A}{v_B} = \frac{3}{4} \text{ or } \frac{\sqrt{\dfrac{\gamma_A RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \frac{3}{4} \qquad (\because l_A = l_B)$$

$$\text{or } \sqrt{\frac{\gamma_A}{\gamma_B}}\sqrt{\frac{M_B}{M_A}} = \frac{3}{4}(\text{as } T_A = T_B) \Rightarrow \frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\left(\frac{16}{9}\right)$$

$$= \left(\frac{5/3}{7/5}\right)\left(\frac{16}{9}\right) \qquad \left(\gamma_A = \frac{5}{3} \text{ and } \gamma_B = \frac{7}{5}\right)$$

$$\therefore \frac{M_A}{M_B} = \left(\frac{25}{21}\right)\left(\frac{16}{9}\right) = \frac{400}{189}$$

(b) Ratio of fundemental frequency in pipe A and in pipe B, which is now closed at both ends

$$\frac{f_A}{f_B} = \frac{v_A/2l_A}{v_B/2l_B} = \frac{v_A}{v_B} \qquad (\text{as } l_A = l_B)$$

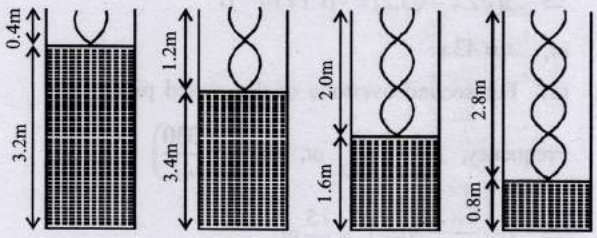
$$= \frac{\sqrt{\dfrac{\gamma_A RT_A}{M_A}}}{\sqrt{\dfrac{\gamma_B RT_B}{M_B}}} = \sqrt{\frac{\gamma_A}{\gamma_B}\cdot\frac{M_B}{M_A}} \qquad (\because T_A = T_B)$$

$$\text{Substituting } \frac{M_B}{M_A} = \frac{189}{400}; \gamma_A = \frac{5}{3} \text{ and } \gamma_B = \frac{7}{3}$$

$$\therefore \frac{f_A}{f_B} = \sqrt{\frac{25}{21}\times\frac{189}{400}} = \frac{3}{4}$$

38. Speed of sound, $v = 340$ m/s.
Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0} = 212.5 \Rightarrow \ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\,\text{m}.$$



In closed pipe only odd harmonics are obtained. Now, let $\ell_1, \ell_2, \ell_3, \ell_4$, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2\,\text{m};$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0\,\text{m}$$

$$7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8\,\text{m};$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6\,\text{m}$$

or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.
Hence heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.
Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$$A = \pi(2\times10^{-2})^2 = 1.26\times10^{-3}\,\text{m}^{-2}$$

$$\text{and } a = \pi(10^{-3})^2 = 3.14\times10^{-6}\,\text{m}^2$$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives,

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

So, rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

$$\Rightarrow \frac{-dH}{dt} = (1.11\times10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$\therefore \quad \frac{dH}{\sqrt{H}} = -1.11\times10^{-2}\,dt$

$\Rightarrow \quad \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = -(1.11\times10^{-2})\int_0^t dt$

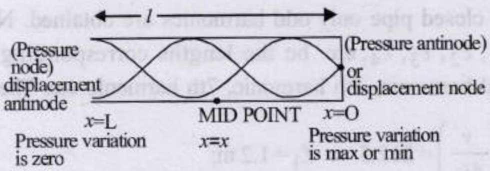
$\Rightarrow \quad 2[\sqrt{2.4}-\sqrt{3.2}] = -(1.1\times10^{-2}).t$

or, $\quad t \approx 43\,s$

39. (a) For second overtone of the closed pipe

Frequency, $f = P\left(\frac{v}{4L}\right)$ or, $440 = 5\left(\frac{330}{4L}\right)$

$\Rightarrow L = \frac{5\times330}{4\times440} \Rightarrow L = \frac{15}{16}$ m.



(b) At any position x, the pressure

$\Delta P = \Delta P_0 \cos kx \cos \omega t$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos \frac{2\pi}{\lambda}x$

For $x = \frac{L}{2} = \frac{15}{2\times16} = \frac{15}{32}$ m (mid point)

Amplitude $= \Delta P_0 \cos\left[\frac{2\pi}{(330/440)}\times\frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure $\because \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end of pipe

Maximum Pressure, $P_{max} = P_0 + \Delta P_0$

Minimum Pressure, $P_{min} = P_0 - \Delta P_0$

40. (i) $y = 4\sin\frac{\pi x}{15}\cos(96\pi t)$

Here amplitude, $A = 4\sin\left(\frac{\pi x}{15}\right)$

At $x = 5$ cm

Amplitude or maximum displacement,

$A = 4\sin\left(\frac{\pi\times5}{15}\right) = 4\times0.866 = 3.46$ cm

(ii) Nodes are the position where $A = 0$

$\therefore \quad \sin\left(\frac{\pi x}{15}\right) = 0 = \sin n\pi \quad \therefore \quad x = 15\,n$

where $\quad n = 0, 1, 2 \quad\quad x = 15$ cm, 30 cm, 60 cm, ...

(iii) At $x = 7.5$ cm, $t = 0.25$ cm velocity of particle

$v = \frac{dy}{dt} = 4\sin\left(\frac{\pi x}{15}\right)[-96\pi\sin(96\pi t)]$

$v = 4\sin\left(\frac{\pi\times7.5}{15}\right)[-96\pi\sin(96\pi\times0.25)]$

$= 4\sin\left(\frac{\pi}{2}\right)[-96\pi\sin(24\pi)] = 0$

(iv) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos[96\pi t]$

$= 2\left[2\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$

$= 2\left[\sin\left(96\pi t+\frac{\pi x}{15}\right)-\sin\left(96\pi t-\frac{\pi x}{15}\right)\right]$

$= 2\sin\left(96\pi t+\frac{\pi x}{15}\right)-2\sin\left(96\pi t-\frac{\pi x}{15}\right)$

$= y_1 + y_2$

Hence components waves

$y_1 = 2\sin\left(96\pi t+\frac{\pi x}{15}\right)$

and $\quad y_2 = -2\sin\left(96\pi t-\frac{\pi x}{15}\right)$

41. Tube open at both ends

Length of tube $l = 48$ cm $= 0.48$m fundamental frequency, $f = 320$Hz velocity of sound in air $v = 320$ m/s

$v = \frac{v}{2(\ell+0.6D)} \quad \therefore \quad 320 = \frac{320}{2(0.48+0.6\times D)}$

$0.48+0.6D = 0.5 \quad \Rightarrow 0.6D = 0.02$

$\Rightarrow \quad D = \frac{0.02}{60}\times100$ cm $= 3.33$ cm (Diameter of tube)

Tube closed at one end

Frequency, $v = \frac{v}{4(\ell+0.3D)} = \frac{320}{4(0.48+0.3\times0.033)} \approx 163$ Hz

Topic-3: Beats, Interference and Superposition of Waves

1. (a) With increase in tension, the frequency produced by string increases. As the beats/sec decreases therefore frequency of tuning fork

$f = 3\left(\frac{v}{4l}\right)+4 = 3\left(\frac{340}{4\times0.75}\right)+4 = 344$Hz

2. (b)

3. (b) Given equation, $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$

$= 2\left(2\cos^2\frac{t}{2}\sin1000t\right)$

or, $y = 2[\cos t+1]\sin1000t$ ...(1)

or, $y = 2\cos t\sin1000t + 2\sin1000t$ ...(2)

or, $y = \sin1001t + \sin999t + 2\sin1000t$ ...(3)

Hence, for the given periodic motion, three independent harmonic motions are superposed.

4. (5) Here the phase difference between the two waves,

$\phi = \pi/2$

Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\frac{\pi}{2}} = \sqrt{16+9+0} = 5$

Topic-4: Musical Sound and Doppler's Effect

1. (a)

2. (b) f_1 = frequency of the police car heard by motorcyclist, f_2 = frequency of the siren heard by motorcyclist.

$f_1 = \frac{330-v}{330-22} \times 176;\ f_2 = \frac{330+v}{330} \times 165;$

$\because\ f_1 - f_2 = 0 \therefore v = 22$ m/s

3. (b) Using the formula $f' = f\left(\frac{v_A + v}{v}\right)$

$\frac{v_A + v}{v} = \frac{5.5}{5}$ and $\frac{V_B + V}{V} = \frac{6}{5} \therefore \frac{v_B}{v_A} = 2$

4. (200) Using Doppler's effect Frequency received by observer

$f_0 = \left(\frac{C \pm V_0}{C \pm V_s}\right) f_s$, C = speed of sound

Case -1: $f_1 = \left(\frac{C+V}{C-V}\right) f_s$

$\Rightarrow\ 288 = \left(\frac{C+V}{C-V}\right) 240$ ..... (i)

Case-2: $f_2 = \left(\frac{C-V}{C+V}\right) f_s$

or, $n = \left(\frac{C-V}{C+V}\right) 240$ ..... (ii)

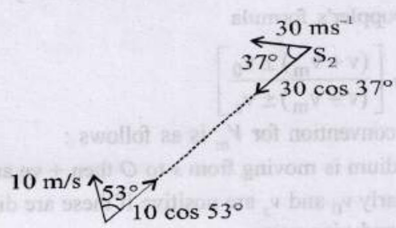
multiplying eq. (i) & (ii)

$(288)(n) = (240)(240)$

$\therefore\ n = \frac{240 \times 240}{288} = 200$

5. (8.13) Apparent frequency heard by observer due to source S_1

$v_1 = v\left[\frac{v + v_0}{v}\right] = 120\left[\frac{330+10}{330}\right] = 120 \times \frac{34}{33} = 123.636$ Hz



Apparent frequency heard by observer due to source S_2

$v_2 = v\left[\frac{v + v_0}{v - v_s}\right] = 120\left[\frac{330 + 10\cos 53°}{330 - 30\cos 37°}\right]$

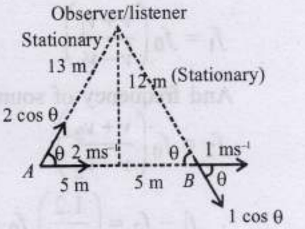
$\therefore v_2 = 120\left[\frac{330 + 10 \times 0.6}{330 - 30 \times 0.8}\right] = 120\left[\frac{336}{306}\right] = 131.764$ Hz

$\therefore$ Beat frequency, $v_b = v2 - v1\ 131.764 - 123.636 = 8.125$ Hz

≈ 8.13 Hz

6. (5.00) Apparent frequency at O due to source at – A

$v_A = v\left[\frac{v}{v - 2\cos\theta}\right]$



Apparent frequency at 'O' due to source at – B

$v_B = v\left[\frac{v}{v + 1\cos\theta}\right]$

$\therefore$ Beat frequency, $v_b = v\left[\frac{v}{v - 2\cos\theta}\right] - v\left[\frac{v}{v + \cos\theta}\right]$

$= v\,v\left[\frac{1}{v - 2\cos\theta} - \frac{1}{v + \cos\theta}\right]$

$= 1430 \times 330\left[\frac{1}{330 - 2 \times \frac{5}{13}} - \frac{1}{330 + \frac{5}{13}}\right]$

$= 1430 \times 330 \times 13\left[\frac{1}{330 \times 13 - 10} - \frac{1}{330 \times 13 + 5}\right]$

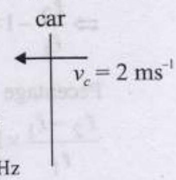
$= 1430 \times 330 \times 13\left[\frac{1}{4280} - \frac{1}{4295}\right] \approx 5$ Hz

7. (6) Frequency observed at car

$\upsilon_1 = \upsilon_0\left(\frac{v + v_c}{v}\right)$

Frequency of reflected sound as observed at the source

$\upsilon_2 = \upsilon_1\left(\frac{v}{v - v_c}\right) = \upsilon_0\left(\frac{v + v_c}{v - v_c}\right)$



$\therefore$ Beat frequency $= \upsilon_2 - \upsilon_0$

$= \upsilon_0\left[\frac{v + v_c - v + v_c}{v - v_c}\right] = \upsilon_0\left[\frac{v + v_c - v + v_c}{v - v_c}\right]$

$= \upsilon_0\left[\frac{2v_c}{v - v_c}\right] = 492\left[\frac{2 \times 2}{330 - 2}\right] = \frac{492 \times 4}{328} = 6$ Hz

8. (3) Resultant of amplitude, Ar

$= \sqrt{I_0}\left[\sin O + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$

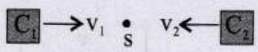
$= \sqrt{I_0}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right] = \sqrt{3}\sqrt{I_0}$

$\therefore \quad I_R = A_R^2 = 3I_0 \quad \therefore \quad n = 3$

9. (7) Firstly, car will be treated as an observer which is approaching the source. Then it will be treated as a source, which is moving in the direction of sound.

Frequency of sound reflected by the car C_1

$f_1 = f_0\left(\frac{v+v_1}{v-v_1}\right)$



And frequency of sound reffected by the car C_2

$f_2 = f_0\left(\frac{v+v_2}{v-v_2}\right)$

$\therefore \ f_1 - f_2 = \left(\frac{1.2}{100}\right) f_0 = f_0\left[\frac{v+v_1}{v-v_1} - \frac{v+v_2}{v-v_2}\right]$

or $\left(\frac{1.2}{100}\right) f_0 = \frac{2v(v_1-v_2)}{(v-v_1)(v-v_2)} f_0$

$(v-v_1) = (v-v_2) \approx v$ as v_1 and v_2 are very very less than v.

$\therefore \left(\frac{1.2}{100}\right) f0 = \frac{2(v_1-v_2)}{v} f_0$

or $(v_1 - v_2) = \frac{v - 1.2}{200} = \frac{330 \times 1.2}{200} = 1.98\,\text{ms}^{-1} = 7\,\text{kmh}^{-1}$

10. **(0.62 to 0.63)** Let ℓ_1 = initial length of pipe

ℓ_2 = new length of pipe

V_T = Speed of tuning fork

In closed organ pipe, $f = \frac{V}{4\ell_1}$

When tuning fork is moved, $f' = f\left(\frac{V}{V - V_T}\right) = \frac{V}{4\ell_2}$

$\Rightarrow \frac{V}{4\ell_1}\left(\frac{V}{V-V_T}\right) = \frac{V}{4\ell_2} \Rightarrow \frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$

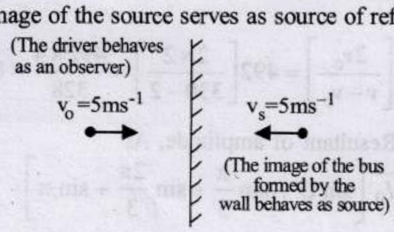
$\Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1 \Rightarrow \frac{\ell_2 - \ell_1}{\ell_1} = \frac{-V_T}{V}$

Pecentage change required in the length of the pipe

$\frac{\ell_2 - \ell_1}{\ell_1} \times 100 = \frac{-2}{320} \times 100 = -0.625\%$

Hence, smallest value of percentage change required in the length of pipe is 0.625%

11. The observer and source are moving towards each other. The image of the source serves as source of reflected sound.

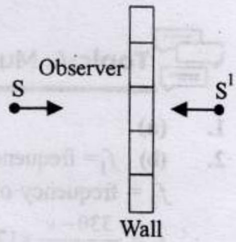


The frequency of sound reflected from the wall

$v' = v\left[\frac{v+v_0}{v-v_s}\right] \Rightarrow v' = 200\left[\frac{342+5}{342-5}\right] \simeq 206$ Hz.

$\therefore$ Frequency of beats $= v' - v = 206 - 200 = 6$ Hz.

12. **False,** if the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound.



Thus in both the cases, one sound coming directly from the source and the other coming after reflection will have the same apparent frequency. Hence, no beats will be heard.

13. **False,** intensity or loudness of sound, $I = \frac{1}{2}\rho V \omega^2 A^2$

Also intensity varies as distance from the point source as $I \propto \frac{1}{r^2}$

As none of the parameters are changing in case of a clear night or a clear day, so the intensity will remain the same.

14. **(648)** Given: velocity of sound V = 324 m/s

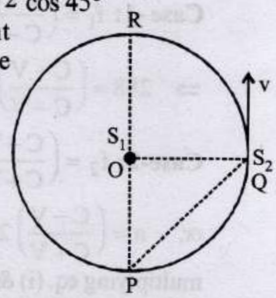
Frequency of source $f_0 = 656$ Hz

Velocity of source S_2, $V_s = 4\sqrt{2}\cos 45°$

From Doppler's effect, apparent frequency measured by the detector, $f = \left(\frac{V}{V+V_s}\right) f_0$

$= \frac{324}{324 + 4\sqrt{2} \times \frac{1}{\sqrt{2}}} \times 656$

$= 648$ Hz



15. **(8.2)** When source S_2 is at R and S_1 approaches the detector with a speed $V_s = 4$ m/s then apparent frequency heard

$f' = \left(\frac{V}{V - V_s}\right) f_0 = \left(\frac{324}{324-4}\right) \times 656 = 664.2$ Hz

16. **A → q; B → p; C → r**

(A)	Pitch	q.	frequency
(B)	quality	p.	waveform
(C)	loudness	r.	intensity

17. By Doppler's formula

$v' = v\left[\frac{(v+v_m) \pm v_0}{(v+v_m) \pm v_s}\right]$

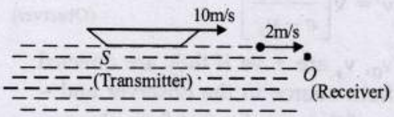
Sign convention for V_m is as follows :

If medium is moving from s to O then + ve and vice versa.

Similarly v_0 and v_s are positive if these are directed from S to O and vice versa.

(a) Velocity of sound in water $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}}$

$= 1445$ m/s



Frequency of sound in water

$v = \frac{v_{water}}{T_{water}} = \frac{1445}{14.45 \times 10^{-3}} = 10^5$ Hz

Frequency is independent of medium $\therefore v_{water} = v_{air}$

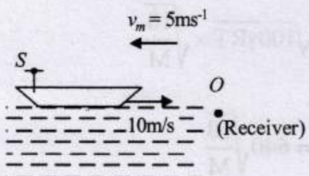
$v_m = +2$ m/s; $v_0 = 0$; $v_s = 10$ m/s

Applying Doppler's formula, $v' = v\left[\frac{v + v_m - v_0}{v + v_m - v_s}\right]$

$\therefore v' = (1 \times 10^5)\left[\frac{1445 + 2 - 0}{1445 + 2 - 10}\right] \therefore v' = 1.007 \times 10^5$ Hz

(b) In air velocity of sound

$= \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4) \times (8.31) \times 293}{28.8 \times 10^{-3}}} = 344$ m/s



Applying Doppler's formula $v' = v\left(\frac{v - v_m - v_0}{v - v_m - v_s}\right)$

$\therefore v' = (1 \times 10^5)\left[\frac{344 - 5 - 0}{344 - 5 - 10}\right] = 1.03 \times 10^5$ Hz

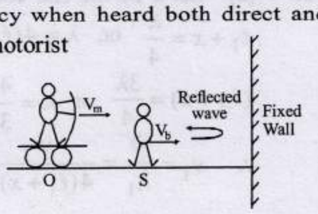
18. The motorist hears the beat frequency as he receives two different frequencies one directly from the sound source or band f and other reflected from the wall.

Apparent frequency when direct sound of band is heard by the motorist

$f' = \left(\frac{v + v_m}{v + v_b}\right) f$

Again apparent frequency when heard both direct and reflected sound by the motorist

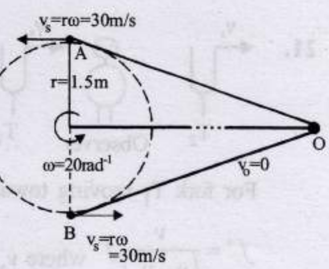
$f'' = \left(\frac{v + v_m}{v - v_b}\right) f$



Hence beat frequency heard by motorist,

$f_b = f'' - f' = \left(\frac{v + v_m}{v + v_b}\right) f - \left(\frac{v + v_m}{v - v_b}\right) f$ or, $f_b = \frac{2v_b (v + v_m)}{v^2 - v_b^2}$

19. The whistle which is emitting sound is being rotated in a circle. The observer is at large distance from the whistle i.e., O



Given: $r = 1.5$ m

and $\omega = 20$ rads^{-1}

Speed of source,

$v_s = r\omega = 1.5 \times 20$

$= 30$ ms^{-1}

When the source is at the position A, then the frequency heard by the observer will be maximum

$v' = v\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330 - 30}\right] = 484$ Hz

When the source is at the position B, then the frequency heard by the observer will be minimum

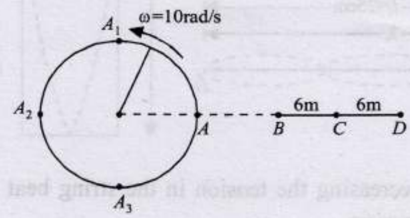
$v'' = v\left[\frac{v}{v + v_s}\right] = 440\left[\frac{330}{330 + 30}\right] = 403.3$ Hz

Hence the range of frequencies heard by the observer $= 403.3$ Hz to 484 Hz.

20. The angular frequency of the detector, $\omega = 2\pi v = 2\pi \times \frac{5}{\pi}$

$= 10$ rad / s

The angular frequency of the detector and the source of sound are equal.



$\Rightarrow$ When the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

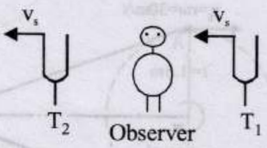
$v_{min} = v\left[\frac{v - v_0}{v + v_s}\right] = 340 \times \frac{(340 - 60)}{(340 + 30)} = 257.3$ Hz

$(\because v_0 = A\omega = 6 \times 10 = 60$ m/s

$v_s = R\omega = 3 \times 10 = 30$ m/s)

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$v_{max} = v\left[\frac{v + v_0}{v - v_s}\right] = 340 \times \frac{(340 + 60)}{(340 - 30)} = 438.7$ Hz

21. 

For fork T_1 moving towards observer,

$$f' = \frac{v}{(v - v_s)} f \quad \text{where } v_s = \text{velocity of fork.}$$

For fork T_2 moving away from observer,

$$f'' = \frac{v}{v + v_s} f \quad \text{where } v = \text{velocity of sound.}$$

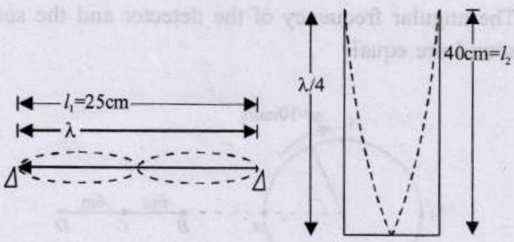
$\therefore$ Beat frequency $= f' - f'' = 3$

$$\text{or } 3 = \frac{vf}{v - v_s} - \frac{vf}{v + v_s} \quad \text{or } 3 = vf\left[\frac{(v+v_s)-(v-v_s)}{(v-v_s)(v+v_s)}\right]$$

$$\text{or } 3 = \frac{vf \times 2v_s}{v^2 - v_s^2} \quad \text{or } 3 = \frac{vf \times 2v_s}{v^2} \text{ as } v_s << v$$

$$\text{or } v_s = \frac{3v^2}{2vf} = \frac{3v}{2f} \text{ or } v_s = \frac{3 \times 340}{2 \times 340} = 1.5 \text{m/s}$$

$\therefore$ Speed of tuning fork = 1.5 m/s

22. 

By decreasing the tension in the string beat frequency is decreasing.

$\therefore$ First overtone frequency of string – fundamental frequency of closed pipe = 8

$$\therefore 2\left(\frac{v_1}{2l_1}\right) - \left(\frac{v_2}{4l_2}\right) = 8 \text{ or } v_1 = l_1\left[8 + \frac{v_2}{4l_2}\right]$$

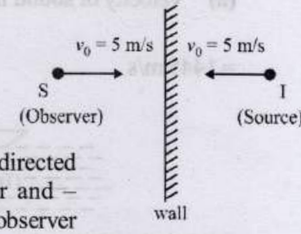
Substituting the value, we have

$$v_1 = 0.25\left[8 + \frac{320}{4 \times 0.4}\right] = 52 m/s$$

$$\text{Now, } v_1 = \sqrt{\frac{T}{\mu}} \therefore T = \mu v_1^2$$

$$\text{or, } T = \left(\frac{m}{l}\right) v_1^2 = \left(\frac{2.5 \times 10^{-3}}{0.25}\right)(52)2 = 27.04 \text{N}$$

23. If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source (I) in the reflecting surface will become the source of the reflected sound.



$$v' = v\left[\frac{c - v_0}{c - v_s}\right]$$

v_0, v_s are + ve if they are directed from source to the observer and – ve if they are directed from observer to source.

$$v' = 256\left[\frac{330 - (-5)}{330 - 5}\right] = 264 \text{ Hz}$$

$\therefore$ Beat frequency $= v' - v = 264 - 256 = 8$

Topic-5: Miscellaneous (Mixed Concepts) Problems

1. **(d)** Here, $\nu = \frac{v}{4l} = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} \times \frac{1}{4l} \Rightarrow v = \nu\lambda = \nu \times 4l$

$\Rightarrow \quad \nu(244) \times 4 \times l = 336.7$ m/s to 346.5 m/s $\quad [\because l = 0.350 \pm 0.005]$

For monatomic gas $\gamma = 1.67$

$$\therefore v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}$$

$$= \sqrt{167RT} \times \sqrt{\frac{10}{M}} = 640\sqrt{\frac{10}{M}}$$

For Neon $M = 20 \therefore v = 640 \times \frac{7}{10} = 448 \text{ ms}^{-1}$

For Argon $M = 36, \quad \therefore v = 640 \times \frac{17}{32} = 340 \text{ ms}^{-1}$

For diatomic gas $\gamma = 1.4$

$$v = \sqrt{140RT}\sqrt{\frac{10}{M}} = 590 \times \sqrt{\frac{10}{M}}$$

For Oxygen $M = 32 \therefore v = 590 \times \frac{9}{16} = 331.87 \text{ ms}^{-1}$

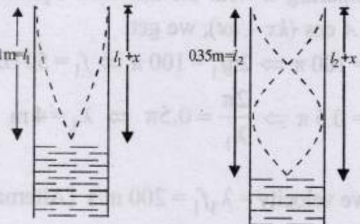
For Nitrogen $M = 28 \quad \therefore v = 590 \times \frac{3}{5} = 354 \text{ ms}^{-1}$

2. **(b)** Let x be the end correction.

$$\ell_1 + x = \frac{\lambda}{4} \quad \text{or,} \quad \lambda = 4(\ell_1 + x)$$

$$(\ell_2 + x) = \frac{3\lambda}{4} \quad \text{or} \quad \lambda = \frac{4}{3}(\ell_2 + x)$$

$$\therefore \quad v_1 = \frac{v}{\lambda_1} = \frac{v}{4(\ell_1 + x)} \quad \therefore v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(\ell_2 + x)}$$



Given $v_1 = v_2 \therefore \dfrac{v}{4(\ell_1+x)} = \dfrac{3v}{4(\ell_2+x)}$ or, $x = 0.025$ m

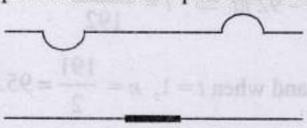
3. (c) Energy, $E \propto A^2\nu^2$ where A = amplitude and ν = frequency.
Also $\omega = 2\pi\nu \Rightarrow \omega \propto \nu$

Experiment = 1 : Amplitude = A and $\nu_1 = \nu$

Experiment = 2 : Amplitude = A and $\nu_2 = 2\nu$

$$\frac{E_2}{E_1} = \frac{A^2\nu_2^2}{A^2\nu_1^2} = 4 \therefore E_2 = 4E_1$$

4. (b) The speed of each pulse is 2cm/s and initially two pulses are 8 cm apart and moving towards each other.



After two seconds pulses will overlap each other.
According to superposition principle the string will not have any distortion and will be straight.
Hence there will be no P.E. The total energy will be only kinetic.

5. (a) Velocity of sound by a stretched string $v = \sqrt{\dfrac{T}{\mu}}$

From Hooke's law, tension in string $T \propto$ extension x

$$\therefore \frac{v}{v'} = \sqrt{\frac{T}{T'}} \quad v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22\,v$$

6. Speed of sound waves in water is greater than in air, so water behave as rarer medium and hence sound wave will be refract into water away from the normal.

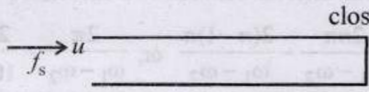
7. (a, d) From Doppler's effect

$$f = f_s\left(\frac{v}{v-u}\right)$$

As the pipe is closed at one end $\quad\therefore$ For resonance,

$$f = f_0\left(\frac{v}{v-u}\right) = nf_0 \text{ where } n = \text{odd integer}$$

(a) For, $u = 0.8v$ and $f_s = f_0$,



$$f = f_0\left(\frac{v}{v-0.8v}\right) = 5f_0$$

(b) For, $u = 0.8v$ and $f_s = 2f_0$, $\Rightarrow f = 2f_0\left(\dfrac{v}{v-0.8v}\right) = 10f_0$

(c) For, $u = 0.8v$ and $f_s = 0.5f_0$

$$f = 0.5f_0\left(\frac{v}{v-0.8v}\right) = 2.5f_0$$

(d) For $u = 0.5v$ and $f_s = 1.5f_0$

$$f = 1.5f_0\left(\frac{v}{v-0.5v}\right) = 3f_0$$

8. (a, b, c)

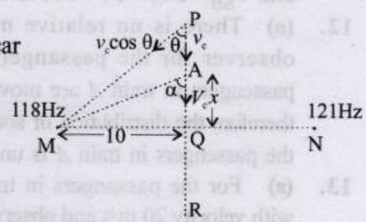
(a) $\nu_P = (V_n - V_m)\left[\dfrac{v + v_c\cos\theta}{v}\right] = 121 - 118\left[\dfrac{v + v_c\cos\theta}{v}\right]$

$\nu_Q = (\nu_N - \nu_M) = 121 - 118 = 3$

$\nu_R = (\nu_N - \nu_M)\left[\dfrac{v + v_c\cos\theta}{v}\right] = (121-118)\left[\dfrac{v - v_c\cos\theta}{v}\right]$

$\therefore \nu_P + \nu_R = 2\nu_Q$

In general when the car is passing through A



$\nu = 3\left[\dfrac{v + v_c\cos\alpha}{v}\right]$ ...(i)

$\therefore \dfrac{d\nu}{d\alpha} = -3\left[\dfrac{v_c\sin\alpha}{v}\right]$ $\left|\dfrac{d\nu}{d\alpha}\right|$ is max when $\sin\alpha = 1$

i.e., $\alpha = 90°$ (at Q)

From eq. (i) $\dfrac{d\nu}{dt} = \dfrac{3v_c}{v}(-\sin\alpha)\dfrac{d\alpha}{dt}$ ...(ii)

Also, $\tan\alpha = \dfrac{10}{x} \therefore \sec^2\alpha\dfrac{d\alpha}{dt} = -\dfrac{10}{x^2}\dfrac{dx}{dt}$

$\therefore \dfrac{d\alpha}{dt} = \dfrac{-10v}{x^2\sec^2\alpha}$ ...(iii)

From eq. (ii) & (iii)

$$\frac{d\nu}{dt} = -\frac{3v_c}{v}\sin\alpha \times \left(\frac{-10v}{x^2\sec^2\alpha}\right) = \frac{30V_c\sin\alpha}{x^2\sec^2\alpha}$$

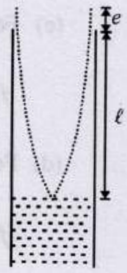
$\therefore \dfrac{d\nu}{dt} = \dfrac{30v_c\sin\alpha}{(10\cot\alpha)^2\sec^2\alpha} = 0.3v_c\sin^3\alpha$. At $\alpha = 90°$

$\dfrac{d\nu}{dt} = max$

9. **(b, d)** When a sound pulse is reflected from open end of a pipe, phase changes by 180°. A high pressure pulse i.e., compression is reflected as a low pressure pulse i.e., rarefaction. When sound pulse is reflected through a rigid boundary (closed end of a pipe), no phase change occurs so a high pressure pulse is reflected as a high pressure pulse.

10. **(a, d)**

The length of the air column at the first resonance is somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air due to end correction (e).



$$\ell + e = \frac{\lambda}{4} \Rightarrow \ell = \frac{\lambda}{4} - e$$

Hence at second resonance the length of air column is more as compared to first resonance. Now, longer the length of air column, more is the absorption of energy and lesser is the intensity of sound heard.

11. **(b)** The speed of sound depends on the frame of reference of the observer.

$V_{SA} = 340 + 20 = 360$ m/s

and $V_{SB} = 340 - 30 = 310$ m/s

12. **(a)** There is no relative motion between source and observer for the passangers in train A. Since all the passengers in train A are moving with a velocity of 20 m/s therefore the distribution of sound intensity of the whistle by the passengers in train A is uniform.

13. **(a)** For the passangers in train B, source is approaching with velocity 20 m/s and observer is receding with velocity 30 m/s $v' = v_1\left[\frac{v - v_0}{v - v_s}\right] = 800\left[\frac{340-30}{340-20}\right] = 800 \times \frac{31}{32}$

$$v'' = v_2\left[\frac{v - v_0}{v - v_s}\right] = 1120 \times \frac{31}{32}$$

$$\therefore \quad v'' - v' = (1120 - 800) \times \frac{31}{32} = 320 \times \frac{31}{32} = 310\text{Hz}.$$

14. **(a)** Beat frequency $= |f_1 - f_2| = (50 - 46) = 4s^{-1}$

one beat frequency consist of maximum and a minimum. So number of maxima are $f = 4$.

15. **(c)** Wave velocity $= \frac{\omega}{k} = \frac{100\pi}{0.5\pi} = 200$ m/s

16. **(d)** The given equations $y_1 = A\cos(0.5\,\pi x - 100\,\pi t)$ and $y_2 = A\cos(0.46\,\pi x - 92\,\pi t)$ represents two progressive wave travelling in the same direction along x-axis with slight difference in the frequency.

Comparing it with the standard equation $y = A\cos(kx - \omega t)$, we get

$\omega_1 = 100\,\pi \Rightarrow 2\pi f_1 = 100\,\pi \Rightarrow f_1 = 50$ Hz and

$$K_1 = 0.5\,\pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4\text{ m}$$

Wave velocity $= \lambda_1 f_1 = 200$ m/s [Alternatively use $v = \frac{\omega}{K}$]

$\omega_2 = 92\,\pi \Rightarrow 2\pi f_2 = 92\,\pi \Rightarrow f_2 = 46$ Hz

Therefore beat frequency $= f_1 - f_2 = 4$ Hz and

$$K_2 = 0.46\,\pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$$

Wave velocity $= \frac{200}{46} \times 46 = 200$ m/s

At $x = 0$,

$$y_1 + y_2 = (A\cos 10\,\pi t) + (A\cos 92\,\pi t) = 0$$

$$\Rightarrow \cos 100\,\pi t = -\cos 92\,\pi t = \cos(-92\,\pi t)$$

$$= \cos[(2n+1)\pi - 92\,\pi t \Rightarrow t = \frac{2n+1}{192}$$

when $t = 0$, $n = -\frac{1}{2}$ and when $t = 1$, $n = \frac{191}{2} = 95.2$

$\therefore$ In 1 second, $y_1 + y_2 = 0$ at $x = 0$ for 96 times

17. Let the two radio waves be represented by the equations

$y_1 = A\sin 2\pi v_1 t$

$y_2 = A\sin 2\pi v_2 t$

Since, $A_1 = A_2 = A$ and detector is at $x = 0$

The equation of resultant wave according to superposition principle

$y = y_1 + y_2 = A\sin 2\pi v_1 t + A\sin 2\pi v_2 t$

$= A[\sin 2\,\pi v_1 t + \sin 2\pi v_2 t]$

$$= A \times 2\sin\frac{(2\pi v_1 + 2\pi v_2)t}{2}\cos\frac{(2\pi v_1 + 2\pi v_2)t}{2}$$

$= 2A\sin\pi(v_1 + v_2)\,t\cos\pi(v_1 - v_2)\,t$

where the amplitude $A' = 2A\cos\pi(v_1 - v_2)\,t$

Now, intensity $\propto$ (Amplitude)^2

$\Rightarrow \quad I \propto A'^2$

$I \propto 4A^2\cos^2\pi(v_1 - v_2)\,t$

Intensity will be maximum when

$\cos^2\pi(v_1 - v_2)\,t = 1$ or, $\cos\pi(v_1 - v_2)\,t = 1$

or, $\pi(v_1 - v_2)\,t = n\pi$

$$\Rightarrow \frac{(\omega_1 - \omega_2)}{2}t = n\pi \quad \text{or,} \quad t = \frac{2n\pi}{\omega_1 - \omega_2}$$

$\therefore$ Time interval between two successive maxima

$$\text{or,} \quad \frac{2n\pi}{\omega_1 - \omega_2} - \frac{2(n-1)\pi}{\omega_1 - \omega_2} \quad \text{or,} \quad \frac{2\pi}{\omega_1 - \omega_2} = \frac{2\pi}{10^3}\text{ s}$$

Time interval between two successive maximas is $2\pi \times 10^{-3}$ sec

(ii) The detector can detect if resultant intensity $\geq 2A^2$

$\therefore$ Resultant amplitude $\geq \sqrt{2}A$

or, $2A\cos\pi(\nu_1-\nu_2)t \geq \sqrt{2A}$

or, $\cos\pi(\nu_1-\nu_2)t \geq \frac{1}{\sqrt{2}}$ or,

$\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$

The detector lies idle when the values of $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]$ is between 0 and $\frac{1}{\sqrt{2}}$

$\therefore \frac{(\omega_1-\omega_2)t}{2}$ is between $\frac{\pi}{2}$ and $\frac{\pi}{4}$

$\therefore t_1 = \frac{\pi}{\omega_1-\omega_2}$ and $t_2 = \frac{\pi}{2(\omega_1-\omega_2)}$

$\therefore$ The time gap $= t_1 - t_2$

$$= \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2}\times 10^{-3}\text{ s}$$

18. **(a)** Compare the given equation $y_1 = A\cos(ax+bt)$ with the standard equation of a plane progressive wave.

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\nu t\right) \Rightarrow \frac{2\pi}{\lambda} = a \Rightarrow \lambda = \frac{2\pi}{a}$$

Also, $2\pi\nu = b$

$\therefore$ Frequency of incident wave, $\nu = \frac{b}{2\pi}$

(b) The wave is reflected by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave $I' = 0.64\, I$

$\Rightarrow I' \propto A'^2 \Rightarrow 0.64\, I \propto A'^2 \Rightarrow 0.64A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of reflected wave

$y_r = 0.8A\cos(ax - bt + \pi) = -0.8A\cos(ax - bt)$

(c) The resultant wave equation

$$y = y_i + y_r = A\cos(ax+bt) + [-0.8A\cos(ax-bt)]$$

Particle velocity

$$v = \frac{dy}{dt} = -Ab\sin(ax+bt) - 0.8Ab\sin(ax-bt)$$
$$= -Ab[\sin(ax+bt) + 0.8\sin(ax-bt)]$$
$$= -Ab[\sin ax\cos bt + \cos ax\sin bt + 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$$

$v = -Ab[1.8\sin ax\cos bt + 0.2\cos ax\sin bt]$

The maximum velocity will occur when $\sin ax = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore$ $|v_{max}| = 1.8Ab$ Also, $|v_{min}| = 0$

(d) $y = [A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$

$= [0.8A\cos(ax+bt) + 0.2A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$

$= [0.8A\cos(ax+bt) - 0.8A\cos(ax-bt)] + [0.2A\cos(ax+bt)]$

$$= 0.8A\left[-2\sin\left\{\frac{(ax+bt)+(ax-bt)}{2}\right\}\sin\left\{\frac{(ax+bt)-(ax-bt)}{2}\right\}\right] 0.2A\cos(ax+bt)]$$

$\Rightarrow y = -1.6A\sin ax\sin bt + 0.2A\cos(ax+bt)$

where $(-1.6A\sin ax\sin bt)$ is the equation of a standing wave and $0.2A\cos(ax+bt)$ is the equation of travelling wave.

The wave is travelling in $-x$ direction.

Antinodes are the points where the amplitude is maximum,

i.e., $\sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$

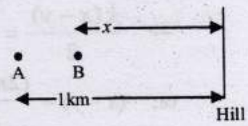
or, $ax = \left[n\pi + (-1)^n\frac{\pi}{2}\right]$ or, $x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$

19. (i) The frequency of the whistle heard by observer on the hill

$$n' = n\left[\frac{v+v_w}{v+v_w-v_s}\right] = 580\left[\frac{1200+40}{1200+40-40}\right] = 599\text{ Hz}$$

(ii) Let echo from the hill is heard by the driver at B which is at a distance x from the hill sound produced by train at a distance 1 km from the hill.

The time taken by the driver to reach from A to B



$$t_1 = \frac{1-x}{40} \quad \text{... (i)}$$

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$$t_2 = \frac{1}{(1200+40)} + \frac{x}{(1200-40)} \quad \text{... (ii)}$$

where t_{AH} = time taken by sound from A to hill with velocity $(1200+40)$

t_{HB} = time taken by sound from hill to B with velocity $1200-40$

From eq. (i) and (ii)

$$t_1 = t_2 \Rightarrow \frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

$\Rightarrow x = 0.935\text{ km}$

The frequency of echo as heard by the driver

$$n'' = n\left[\frac{(v-v_w)+v_s}{(v-v_w)-v_0}\right] = 580\left[\frac{(1200-40)+40}{(1200-40)-40}\right] = 621\text{ Hz}$$

20. (i) When two identical waves travelling in opposite direction superimpose, we get standing waves.

Hence following two equations will produce standing wave

$z_1 = A\cos(kx - \omega t)$

$z_2 = A\cos(kx + \omega t)$

The resultant wave is given by $z = z_1 + z_2$

$\Rightarrow \quad z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$

$= 2A\cos kx\cos\omega t$

The resultant intensity will be zero when

$2A\cos kx = 0$

$$\Rightarrow \cos kx = \cos\frac{(2n+1)}{2}\pi$$

$$\Rightarrow kx = \frac{2n+1}{2}\pi \quad \Rightarrow x = \frac{(2n+1)\pi}{2k} \text{ where } n = 0, 1, 2, ...$$

(ii) The transverse wave z_1 travelling in $+x$-axis and z_3 travelling in $+y$-axis

$z_1 = A\cos(kx - \omega t) \Rightarrow z_3 = A\cos(ky - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive x and positive y axes.

The resultant wave $z = z_1 + z_3$

$z = A\cos(kx - \omega t) + A\cos(ky - \omega t)$

$$\Rightarrow \quad z = 2A\cos\frac{(x-y)}{2}\cos\left[\frac{k(x+y) - 2\omega t}{2}\right]$$

The resultant intensity will be zero when

$$2A\cos\frac{k(x-y)}{2} = 0 \quad \Rightarrow \quad \cos\frac{k(x-y)}{2} = 0$$

$$\Rightarrow \quad \frac{k(x-y)}{2} = \frac{2n+1}{2}\pi$$

$$\text{or,} \quad (x-y) = \frac{(2n+1)}{k}\pi \text{ where n} = 0, \pm 1, \pm 2 \text{ etc.}$$

21. Using, $Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\Delta\ell/\ell} = \frac{T}{\alpha A\Delta\theta} \quad \therefore T = YA\,\alpha\,\Delta\theta$

$(\because \Delta\ell = \ell\alpha\,\Delta\theta \Rightarrow \frac{\Delta l}{l} = \alpha\,\Delta\theta)$

Frequency of fundamental mode of vibration

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{YA\alpha\,\Delta\theta}{m}}$$

$$= \frac{1}{2\times 1}\sqrt{\frac{2\times 10^{11}\times 10^{-6}\times 1.21\times 10^{-5}\times 20}{0.1}} = 11\text{ Hz}$$

22. (a) Using $y = \dfrac{F/A}{\Delta l/l}$ and $\Delta l = l\,\alpha\,\Delta\theta$

$\therefore \quad F = YA\alpha\Delta\theta$

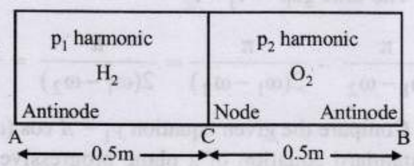
Speed of transverse wave

$$v = \sqrt{\frac{T}{M}}\left[\text{where } \mu = \text{mass per unit length} = \frac{Al\rho}{\ell} = A\rho\right]$$

$$= \sqrt{\frac{YA\alpha\Delta\theta}{A\rho}} = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$$

$$\therefore v = \sqrt{\frac{1.3\times 10^{11}\times 1.7\times 10^{-5}\times 20}{9\times 10^3}} = 70\text{ m/s}$$

23. According to question, the diaphragms A and B are set into vibrations of same frequency. Diaphragm C is a node. So, at A and B antinodes are formed.



$$\frac{v_1}{4\ell}\times p_1 = \frac{v_2}{4\ell}\times p_2 \Rightarrow \frac{p_1}{p_2} = \frac{v_1}{v_2} = \frac{3}{11} \text{ or, } 11p_1 = 3p_2$$

i.e., The third harmonic in AC is equal to 11th harmonic in CB.

Now, the fundamental frequency in AC

$$= \frac{v_1}{4\ell} = \frac{1100}{4\times 0.5} = 550\text{ Hz}$$

and the fundamental frequency in CB

$$= \frac{v_2}{4\ell} = \frac{300}{4\times 0.5} = 550\text{ Hz}$$

$\therefore$ Frequency in $AC = 3\times 550 = 1650$ Hz

and frequency in $CB = 11\times 150 = 1650$ Hz